Computer code for double beta decay QRPA based calculations

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Abstract. The computer code developed by our group some years ago for the evaluation of nuclear matrix elements, within the QRPA and PQRPA nuclear structure models, involved in neutrino-nucleus reactions, muon capture and $\beta^{\pm}$ processes, is extended to include also the nuclear double beta decay.

INTRODUCTION

The two neutrino double beta ($\beta\beta_{2\nu}$) decay $(A,Z) \rightarrow (A,Z+2)+2e^- + 2\tilde{\nu}$ can occur by successive β decays (see Figure 1 a):

$$\begin{aligned}(A,Z) &\rightarrow (A,Z+1)+e^- + \tilde{\nu} \\ &\rightarrow (A,Z+2)+2e^- + 2\tilde{\nu},\end{aligned} \tag{1}$$

passing through the intermediate virtual states of the $(A,Z+1)$ nucleus (see Figure 2). More interesting is the neutrinoless double beta ($\beta\beta_{0\nu}$) decay which is possible if the neutrino is a Majorana particle, i.e., equal to its own antiparticle (see Figure 1 b):

$$\begin{aligned}(A,Z) &\rightarrow (A,Z+1)+e^- + \tilde{\nu} \equiv (A,Z+1)+e^- + \nu \\ &\rightarrow (A,Z+2)+2e^-.\end{aligned} \tag{2}$$

Also it is interesting the neutrinoless double beta decay with majoron emission ($\beta\beta_{0\nu\chi}$) where a Nambu-Goldstone boson χ (called by majoron) is produced by spontaneous symmetry breaking (see Figure 1 c):

$$(A,Z) \rightarrow (A,Z+2)+2e^- + \chi. \tag{3}$$

It is important to mention that none of the experimental searches on neutrino oscillations give us the neutrino mass so directly as the $\beta\beta_{0\nu}$ mode could give if experimentally observed.

These two modes of disintegration are related through the nuclear structure effects, i.e., $\mathscr{M}_{2\nu}$ and $\mathscr{M}_{0\nu}$ present many similar features and it is well established that we shall not understand the $\beta\beta_{0\nu}$ decay unless we understand the $\beta\beta_{2\nu}$ decay [1, 2].A really interesting point to analyze is the extreme sensitivity of the $\beta\beta_{2\nu}$ decay amplitudes, $\mathscr{M}_{2\nu}$, on the residual interaction in the particle-particle channel. Differently from other QRPA formalisms used in nuclear structure to evaluate $\beta\beta$-decay processes, we are treating the intermediate states without performing a second charge-conserving QRPA to describe the $\beta\beta$-decays to excited final states.

A few years ago, a computer code for quasiparticle random phase approximation (QRPA) and projected quasiparticle random phase approximation (PQRPA) models of nuclear structure was published and explained in details [3].The

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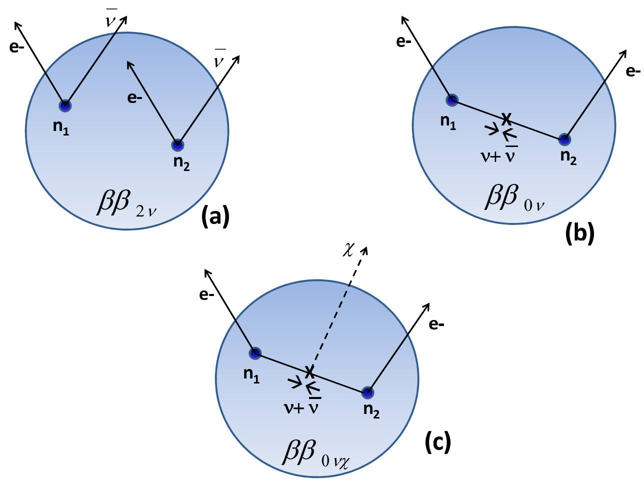

FIGURE 1. Scheme for: a) $\beta\beta_{2\nu}$ decay; b)$\beta\beta_{0\nu}$ decay; c) $\beta\beta_{0\nu\chi}$ decay.

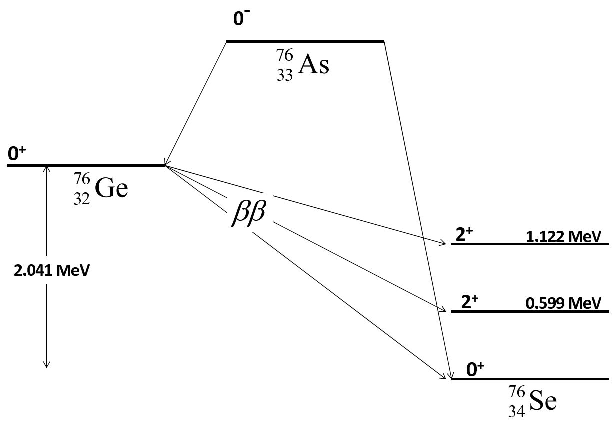

FIGURE 2. Scheme for $\beta\beta$-decay in ^{76}Ge.

code name adopted was QRAP ((Q)uasiparticle (RA)ndom (P)hase approximation). In the QRAP code, the residual interaction is approximated by a simple δ-force. The main application of the code consists in evaluating nuclear matrix elements (NMEs) involved in neutrino-nucleus reactions, as for example in electronic neutrino cross sections, muon capture, β^+ and β^- decay rates. In this work we implemented the nuclear double beta decay ($\beta\beta$-decay) in the code, which will be rebaptized as QRAP-$\beta\beta$. The actual code summarizes and gives a new fashion of the formalism presented in Refs. [4, 5] for $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decays, which was based on the Fourier-Bessel expansion of the weak Hamiltonian adapted for nuclear structure calculations.

$\beta\beta$ DECAY HALF-LIVES AND NUCLEAR MATRIX ELEMENTS

For $0^+ \to 0^+$ transitions, the inverse half-lives and the NMEs of $\beta\beta_{2\nu}$, $\beta\beta_{0\nu}$ and $\beta\beta_{0\nu\chi}$ decays are related as [1]:

$$T_{1/2}^{-1} = \mathscr{G}\mathscr{F}^2|\mathscr{M}|^2, \qquad \mathscr{F} = \begin{cases} 1 & \text{for} \quad \beta\beta_{2\nu} \\ \langle m_\nu \rangle / m_e & \text{for} \quad \beta\beta_{0\nu} \\ \langle g_M \rangle & \text{for} \quad \beta\beta_{0\nu\chi} \end{cases}, \tag{4}$$

where $\mathscr{G}$ is the kinematical factor which depends on the corresponding phase space, $\mathscr{M}$ is the NME and $\langle m_\nu \rangle$, m_e and $\langle g_M \rangle$ in $\mathscr{F}$ are the neutrino effective mass, the electron mass and the majoron-neutrino effective coupling constant, respectively. $\mathscr{M}_{2\nu}$, $\mathscr{M}_{0\nu}$ and $\mathscr{M}_{0\nu\chi}$ show very similar features. To get confidence in the nuclear model used to evaluate theoretically the NME, the procedure is to find an agreement between experimental and theoretical values for $\mathscr{M}_{2\nu}$, and then to use the same nuclear model (and parametrization) to describe consistently $\mathscr{M}_{0\nu}$ and $\mathscr{M}_{0\nu\chi}$.

By this reason, we start analyzing in this work the $\beta\beta_{2\nu}$ matrix element. When only allowed transitions are considered, the matrix element corresponding to $|0^+_f\rangle$ final state can be written, independently of the adopted nuclear model, as

$$\mathscr{M}_{2\nu}(f) = \sum_{\lambda=0,1} (-)^\lambda \sum_\alpha \left[\frac{\langle 0^+_f || \mathscr{O}^{\beta^-}_\lambda || \lambda^+_\alpha \rangle \langle \lambda^+_\alpha || \mathscr{O}^{\beta^-}_\lambda || 0^+ \rangle}{\mathscr{D}_{\lambda^+_\alpha, f}} \right] \equiv \mathscr{M}^F_{2\nu}(f) + \mathscr{M}^{GT}_{2\nu}(f), \tag{5}$$

where the summation goes over all intermediate virtual states $|\lambda^+_\alpha\rangle$ and $\mathscr{D}_{\lambda^+_\alpha, f} = E_{\lambda^+_\alpha} - (E_0 + E_{0^+_f})/2$ is the energy denominator with E_0 and $E_{0^+_f}$ being the energy of the initial ($|0^+\rangle$) and final ($|0^+_f\rangle$) states, respectively. The operator for β^--decay is

$$\mathscr{O}^{\beta^-}_\lambda = (2\lambda+1)^{-1/2} \sum_{pn} \langle p || \mathrm{O}_\lambda || n \rangle \left(c^\dagger_p c_{\bar{n}} \right)_\lambda ,$$

with $\mathrm{O}_0 = 1$ for Fermi (F) and $\mathrm{O}_1 = \sigma$ for Gamow-Teller (GT) transitions, being $c^\dagger$ and c the particle creation and annihilation operators. The corresponding operators for β^+-decay are $\mathscr{O}^{\beta^+}_\lambda = \left(\mathscr{O}^{\beta^-}_\lambda \right)^\dagger$. Finally, it is important to mention that in this work we follow the procedure developed in Refs. [1, 6] for the evaluation of the matrix elements $\mathscr{M}^{F,GT}_{2\nu}(f)$, where a Fourier-Bessel expansion of the weak Hamiltonian was performed.

RESULTS AND FINAL REMARKS

Our main purpose is to develop a numerical code for the evaluation of weak observables in nuclear double beta decays. The implementation of such a code, including the three decay modes, demands an amazing effort both theoretically and numerically. Much effort has been spent by our group along last years in order to estimate the NME and the corresponding half-lives [4, 5, 6, 7, 8, 9, 10]. From the pioneering works of Krmpotić et al. [1], we decided to use in those works individual codes separated in three groups according to the considered decay mode. However, it will be very useful for the nuclear community to have at their disposal an unified code containing all the three decay modes together. With this aim, we have started in this work with a first version named as QRAP-2ν v0.0 where we recovered the $\beta\beta_{2\nu}$ with the allowed contributions for the NMEs.

Details for the theoretical formalism to evaluate the $\beta\beta_{2\nu}$ NME with the allowed contributions is developed in Refs. [4, 5, 6]. Relative to the QRPA nuclear calculations, we have employed a residual δ-force $V = -4\pi(v_s P_s + v_t P_t)\delta(r)$, with different strength constants v_s and v_t for the particle-hole, particle-particle and pairing channels [1, 3, 11]. The single-particle energies, as well as the pairing parameters $v^{pair}_s(p)$ and $v^{pair}_s(n)$, have been fixed by fitting the experimental pairing gaps to a Wood-Saxon potential with the procedure described in [1, 3]. In this way, the proton and neutron gap equations have been solved for the intermediate $(N-1, Z+1)$ nucleus as in Ref. [11], and we deal only with one QRPA equation. Note that in this procedure we avoid the problem of overlapping of two sets of the same intermediate states generated from initial and final nuclei.

The schematic flux diagrams for the original QRAP code and the new QRAP-2ν v0.0 version is shown in Figure 3. The numerical results obtained with this code for the GT NMEs in the $\beta\beta_{2\nu}$-decay for ^{100}Mo are shown in Table 1. The value reproduced with the recovered and previously described code for QRPA is underlined. We also compare with other calculations performed within different nuclear models: another QRPA calculation from Ref. [12] (this QRPA

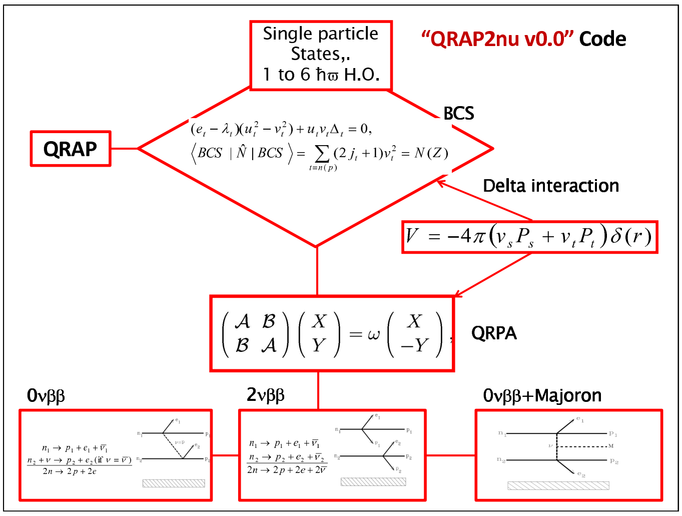

FIGURE 3. Flux diagram for QRAP-2ν v0.0 code.

TABLE 1. $\mathscr{M}_{2\nu}^{GT}$ (in units of MeV^{-1}), $\mathscr{G}_{2\nu}^{GT}$ corresponding to $g_A/g_V = -1.0$ in [1] (in units of $10^{-20}\ yr^{-1}$) and half-life (in units of $10^{19}\ yr$) for $\beta\beta_{2\nu}$ decay of ^{100}Mo to the ground and first excited states of ^{100}Ru.

Model	$0^+ \to 0^+_{g.s.}$	$0^+ \to 0^+_1$
QRPA [1]	0.102	
QRPA [12]	0.101	0.138
FQTDA (I)	0.039	0.059
$\mathscr{M}_{2\nu}^{GT}$	0.358 ± 0.011	0.311 ± 0.041
$T_{2\nu}$	$0.768 \pm 0.002(\text{stat}) \pm 0.054(\text{syst})$ [13]	$60^{+19}_{-11}(\text{stat}) \pm 6(\text{syst})$ [13]
$\mathscr{G}_{2\nu}^{GT}$	387	6.61

solves two RPA equations to calculate the NME) and the FQTDA formalism (Four Quasiparticle Tamm-Damcoff) [2]. In particular, the FTQDA is a new formalism employed to evaluate the NME based on the well-known QTDA [14]. The main difference in comparison to the QRPA comes from the description of the final $(N-2, Z+2)$ nucleus. In the QRPA the final nucleus is a two-quasiparticle excitation state of the vacuum ground state in the bosonic approximation (the BCS wave function) whereas in the FQTDA the final nuclear results in a four-quasiparticle excitation. The experimental data for $\mathscr{M}_{2\nu}^{GT}$ were obtained with the experimental half-life $T_{2\nu}$ from [13] using Eq. (4) with the values $\mathscr{G}_{2\nu}^{GT}$ also shown in Table 1.

Summarizing, we have implemented a numerical code to evaluate $\beta\beta$-decay processes using a formalism based in a Fourier-Bessel expansion of the weak Hamiltonian. This open code could be an open window in real advances in studies of new physics, offering a gold opportunity to the scientific community. When it will be completed, this code will summarize more of thirty years of theoretical and numerical work in $\beta\beta$-decay developed by Prof. F. Krmpotić and collaborators.

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